Comparisons to analytic solution for simple wedge

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WaveQ3D needs a testing benchmark that clearly demonstrates 3-D effects in transmission loss. This whitepaper derives an analytic solution for acoustic transmission loss in the wedge-shaped, 3-D ocean environment illustrated in Figure 1. We examine a scenario in which receivers are at the same distance from the wedge apex as the source, but offset in range across the slope. In an 2-D model, these receivers appear to exist in an environment of constant depth. Because the 3-D solution horizontally refracts acoustic energy down the slope, the 3-D solution has higher transmission loss as a function of range across the slope than the 2-D model. This difference provides a benchmark that clearly demonstrates 3-D transmission loss effects.

# Derivation of analytic solutions

Figure 1 defines the wedge geometry in Cartesian coordinates:

= angle of the wedge relative to the horizontal;

= range of this source and receiver from the wedge apex along the ocean surface;

= depth of this source and receiver down from the ocean surface; and

= cross-slope distance of the receiver relative the vertical source/origin plane.

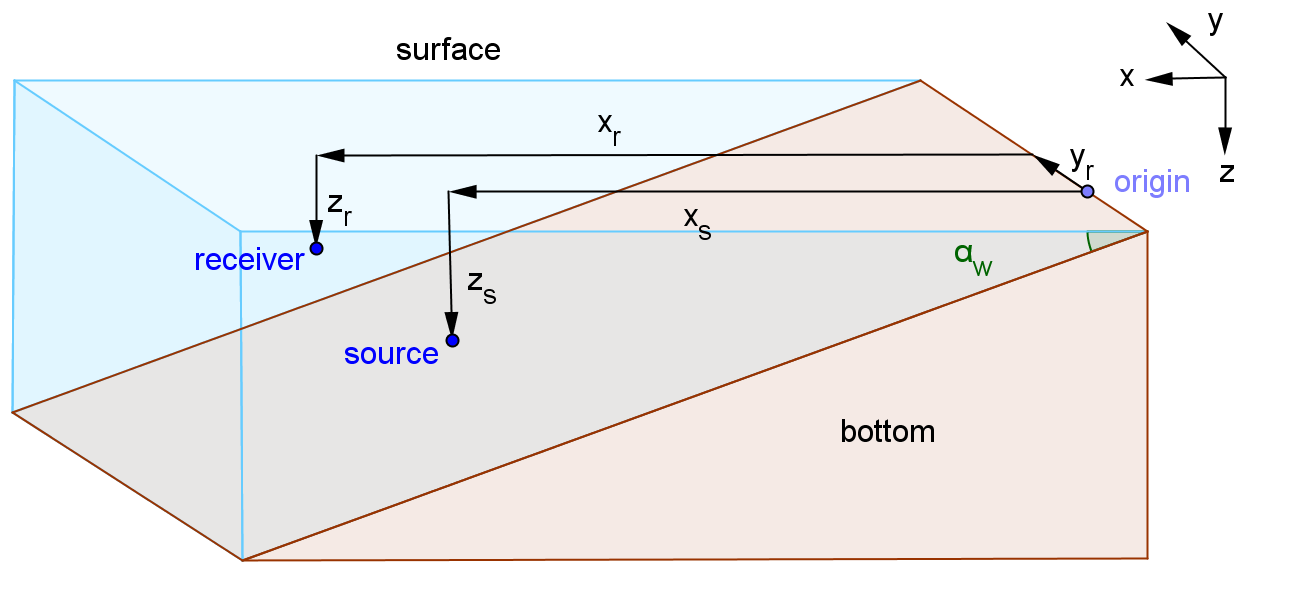


Figure – Acoustic transmission loss geometry for 3-D wedge

Using the method of images, we assume that each reflection gives rise to a source image, and that these images lie on a circle centered on the apex of the wedge. This derivation is very similar to the Deane/Buckingham model defined in reference [1], but it simplifies that model by assuming that interface reflection coefficients are limited to . Figure 2 is a cross-slope view of the 3-D wedge showing each of the image sources and each virtual interface. In this illustration, surface interfaces are shown with a dashed line, bottom interfaces are shown with a dot-dashed line, and source images are shown as dots along the circumference of a circle whose radius defined by the original distance of the source from the apex.

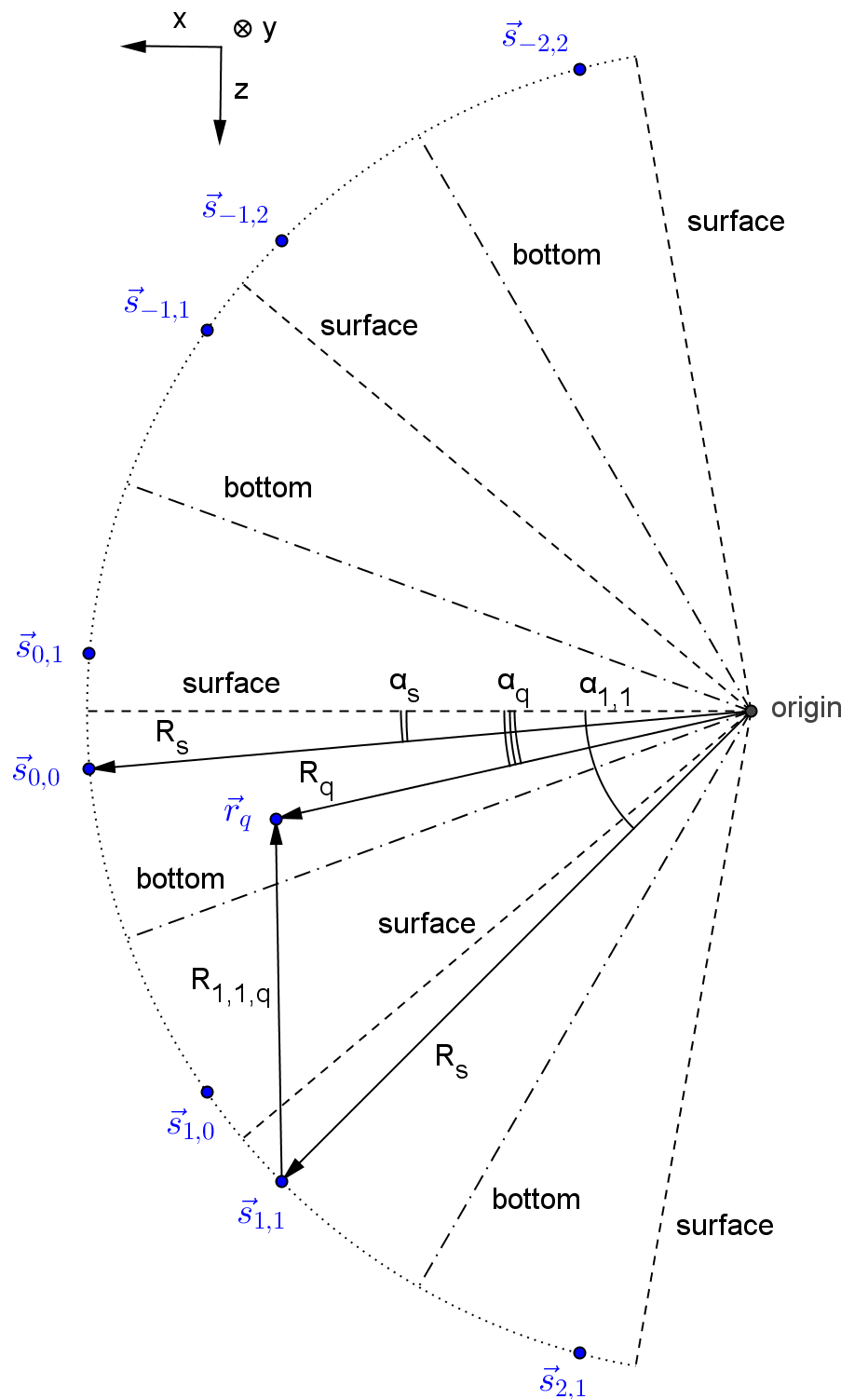


Figure – Geometry for method of images in a 3-D wedge

The complex pressure at each receiver location is a sum of spherical wave contributions from each source image. If we assume that the reflection coefficient is +1 at the bottom and -1 at the surface, this takes the form:

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where

= number of bottom reflections for source image, negative if above surface;

= number of surface reflections for source image, negative if above surface;

= maximum number of bottom bounces;

= location of each source image;

= index number for each receiver;

= location of each receiver;

= slant range from each source image to each receiver;

= speed of sound in water;

= signal frequency;

= acoustic wave number ; and

= total complex pressure for each receiver.

To compute , reference [1] defines a cylindrical coordinate system whose axis travels along the wedge apex:

= slant range of original source from the wedge apex;

= angle of original source down from the ocean surface;

= angle of each source image, relative to the ocean surface, negative if above surface;

= slant range of each receiver from the wedge apex;

= angle of each receiver down from the ocean surface; and

= cross-slope distance of each receiver relative the vertical source/origin plane.

An inspection of the geometry in Figure 2 allow us to compute and for the 3-D wedge.

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Source images outside of the range result in “imaginary” images that contribute to the diffracted component of the acoustic field. Reference [1] states that for small wedge angles and locations far from the apex, the diffracted components are negligible and need not be considered.

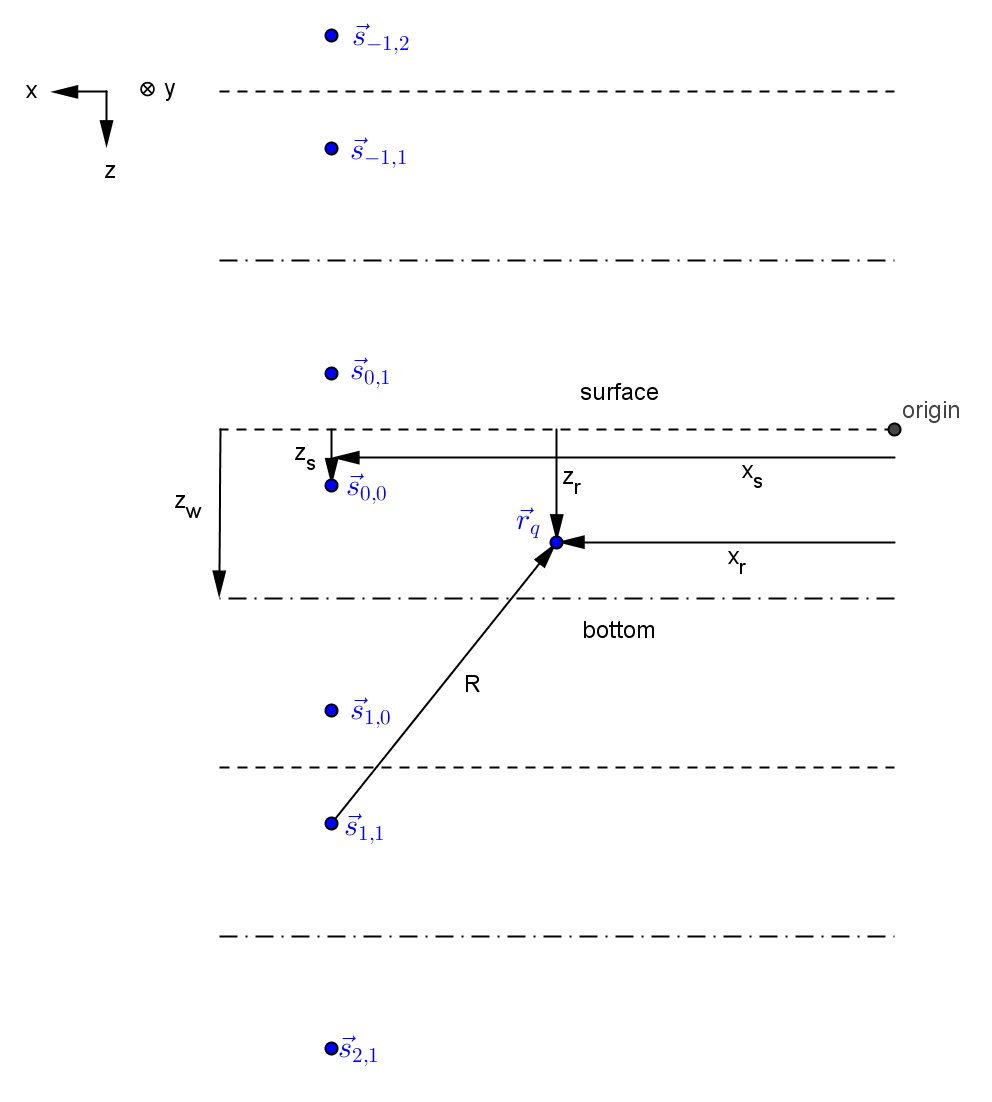


Figure 3 – Method of images for flat bottom

An equivalent solution for an environment of constant depth are derived by inspection of Figure 3.

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| --- | --- | --- |
|  |  | (4) |

|  |  |  |
| --- | --- | --- |
|  |  | (5) |

where

= range and depth of original source relative to ocean surface;

= range and depth of each receiver relative to ocean surface;

= cross-slope distance of the receiver relative the vertical source/origin plane;

= depth of each source image;

= water depth;

Figure 4 compares the analytic solutions for a simplified version of the ASA wedge benchmark [2], to an equivalent 2-D environment of constant depth. To support later comparisons to WaveQ3D, Figure 4 includes results both at the 25 Hz frequency specified by the ASA wedge benchmark, and 2000 Hz, a frequency at which Gaussian beam ray theory should be accurate.

= 2.86o

= 4000 meters

= 100 meters

= 200 meters

= 1500 m/s



Figure – Analytic solutions for small wedge angle

Figure 5 provides a similar comparison for wide angle benchmark from reference [1]. Note that the cross range axis in Figure 5 only extends to 10 km, while the axis in Figure 4 extends to 70 km.

= 21o

= 520 meters

= 100 meters

= 200 meters

= 1500 m/s



Figure – Analytic solutions for wide wedge angle

In both Figure 4 and Figure 5, the 3-D wedge has significantly stronger losses as a function of cross slope range than predicted by an equivalent 2-D model. The physical interpretation of this phenomena is that the 3-D wedge turns high angle paths toward deep water so that they fail to contribute to the received level at longer ranges. At some cutoff range, the 3-D propagation is reduced to a combination of the direct and surface reflected paths, and the transmission loss takes on the characteristics of a Llyod’s Mirror scenario. This cut-off range decreases as the wedge angle increases. Because the 2-D constant depth solution does not reject higher angle paths, it exhibits less transmission loss. In the next section, this difference is used as a benchmark to demonstrate 3-D transmission loss effects in the WaveQ3D model.

# WaveQ3D comparisons to analytic solutions

Because the WaveQ3D calculations are performed in geodetic coordinates, the simple wedge used in our analytic solution can only be approximated in WaveQ3D. On a round Earth, an interface with constant slope is a curved surface instead of a plane. To minimize the impact of this curvature, the wide wedge angle scenario is used to shorten the range over which 3-D effects can be observed. The source and receivers are placed at a depth of 100 meters at the Equator. The water depth at this point is set to 200 meters and the bottom slope is a constant 21o, sloping down to the north, at all latitudes and longitudes. This definition orients the wedge illustration in Figure 1 such that the x-direction is north, the y-direction is east, and the z-direction is down. Receivers are placed east of the source, along the y-direction, at varying cross slope ranges.

Figure 6 and Figure 7 illustrate the propagation of the WaveQ3D ray paths in this scenario.



Figure 6 – 3-D view of individual ray path for -10o D/E and 135o AZ



Figure 7 - Top down view of ray paths for -10o D/E

Figure 8 and Figure 9 compare the WaveQ3D results to the analytic solutions for the flat bottom and wedge analytic solutions. Figure 8 illustrates the coherent solution computed by Eqn. (1); Figure 9 compares the incoherent solutions to emphasis the differences between models.



Figure – Compare coherent WaveQ3D to analytic solutions



Figure - Compare incoherent WaveQ3D to analytic solutions

# Works Cited

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| [1] | G. B. Deane and M. Buckingham, "An analysis of the three-dimensional sound field in a penetrable wedge with a stratified fluid or elastic basement," *J. Acoust. Soc. Am.,* vol. 93, no. 3, pp. 1319-1328, March 1993. |
| [2] | F. B. Jensen and C. M. Ferla, "Numerical solutions of range‐dependent benchmark problems in ocean acoustics," *J. Acoust. Soc. Am.,* vol. 87, no. 4, pp. 1499-1510, 1990. |