Comparisons to analytic solution for simple wedge

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WaveQ3D needs a testing benchmark that clearly demonstrates 3-D effects in transmission loss. This whitepaper derives an analytic solution for acoustic transmission loss in the wedge-shaped, 3-D ocean environment illustrated in Figure 1. We examine a scenario in which receivers are at the same distance from the wedge apex as the source, but offset in range across the slope. In an 2-D model, these receivers would appear to exist in an environment of constant depth. Because the 3-D solution horizontally refracts acoustic energy down the slope, we expect it to have higher transmission loss as a function of range across the slope than the 2-D model. This difference provides the benchmark for clearly demonstrates 3-D effects in transmission loss.

# Derivation of analytic solutions

Figure 1 defines the wedge geometry in Cartesian coordinates:

= angle of the wedge relative to the horizontal;

= range of this source and receiver from the wedge apex along the ocean surface;

= depth of this source and receiver down from the ocean surface; and

= cross-slope distance of the receiver relative the vertical source/origin plane.

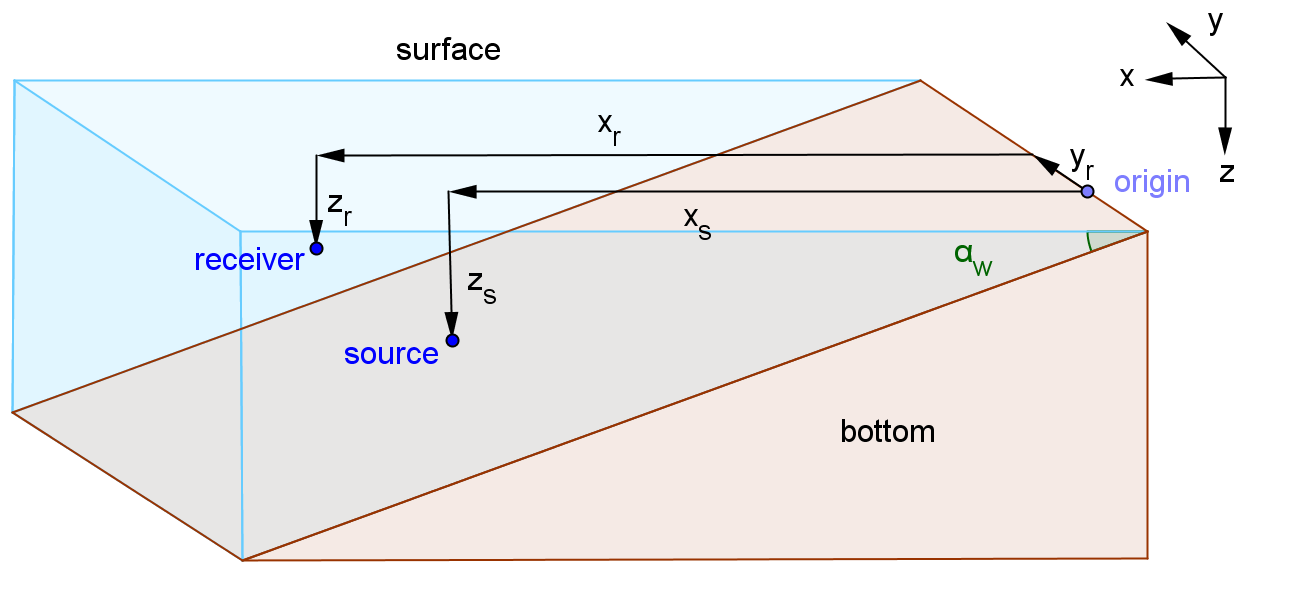


Figure 1 – Acoustic transmission loss geometry for 3-D wedge

Using the method of images, we assume that each reflection gives rise to a source image, and that these images lie on a circle centered on the apex of the wedge. This derivation is very similar to the Deane/Buckingham model defined in reference [1], but it simplifies that model by assuming that the interface reflection coefficients are limited to . Figure 2 is a cross-slope view of this geometry showing each of the image sources and each virtual interface. In this illustration, surface interfaces are shown with a dashed line, bottom interfaces are shown with a dot-dashed line, and source images are shown as dots along the circumference of a circle whose radius defined by the original location of this source.

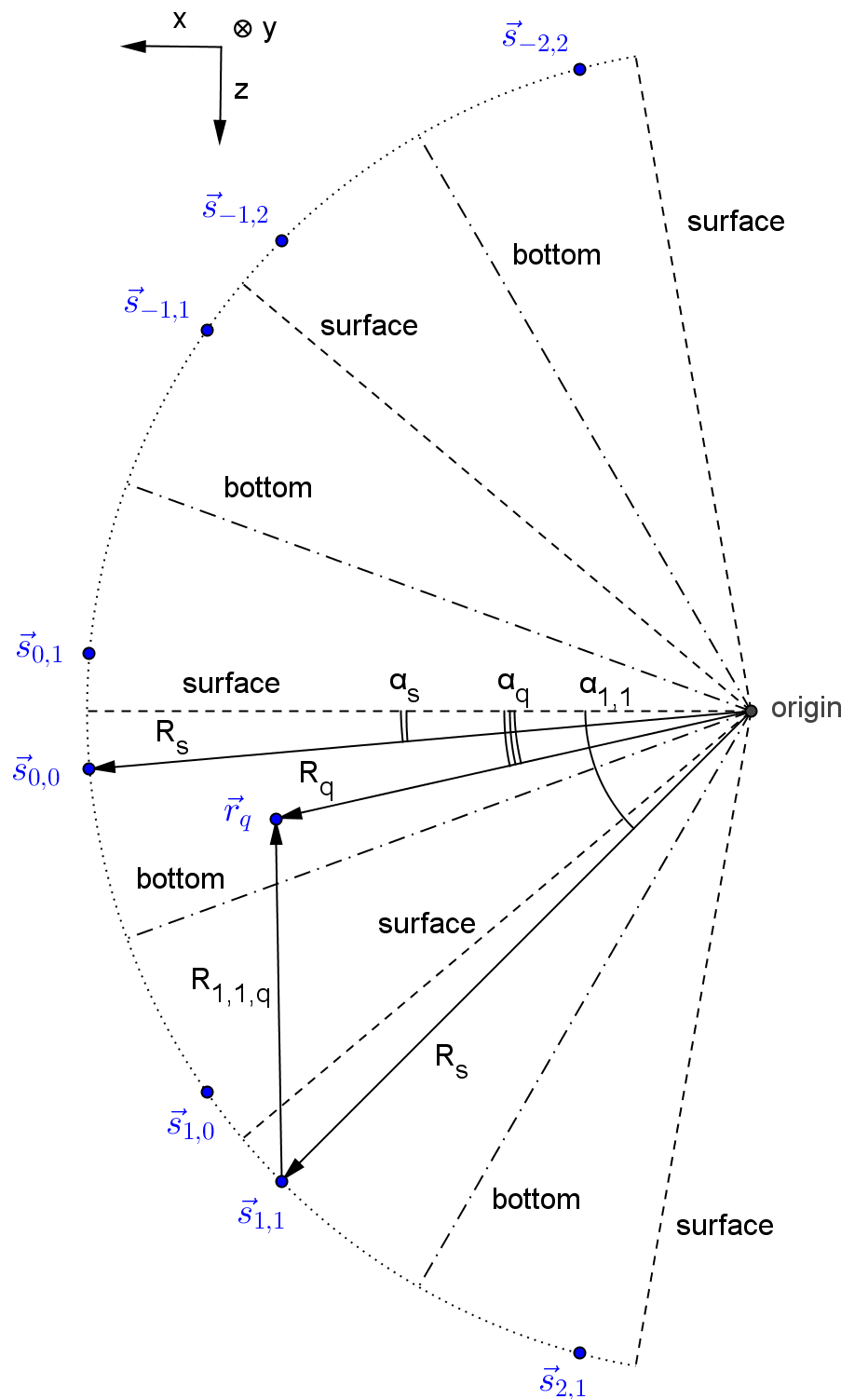


Figure 2 – Geometry for method of images

The complex pressure at each receiver location is a sum of spherical wave contributions from each source image. If we assume that the reflection coefficient is +1 at the bottom and -1 at the surface, this takes the form:

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

where

= number of bottom reflections for source image, negative if above surface;

= number of surface reflections for source image, negative if above surface;

= maximum number of bottom bounces;

= location of each source image;

= index number for each receiver;

= location of each receiver;

= slant range from each source image to each receiver;

= speed of sound in water;

= signal frequency;

= acoustic wave number ; and

= total complex pressure for each receiver.

To compute , we define a cylindrical coordinate system whose axis travels along the wedge apex:

= slant range of original source from the wedge apex;

= angle of original source down from the ocean surface;

= angle of source image down from the ocean surface, negative if above surface;

= slant range of each receiver from the wedge apex;

= angle of each receiver down from the ocean surface; and

= cross-slope distance of each receiver relative the vertical source/origin plane.

An inspection of the geometry in Figure 2 allow us to compute and

|  |  |  |
| --- | --- | --- |
|  |  | (2) |

|  |  |  |
| --- | --- | --- |
|  |  | (3) |

Source images outside of the range result in “imaginary” images that contribute to the diffracted component of the acoustic field. Reference [1] states that for small wedge angles and locations far from the apex, the diffracted components are negligible and need not be considered.

Figure 3 and Figure 4 illustrate the analytic solution for transmission loss (Equations (1) through (3)) as a function of range across the slope , for an “ideal wedge” version of the scale model results discussed in reference [2], scaled to in-ocean values.

= 150 meters

= 20o

= 10o (middle of water column)



Figure 3 – Transmission loss as function of cross-slope range at 70 Hz



Figure 4 – Transmission loss as function of cross-slope range at 110 Hz

In 2-D models, paths across the slope appear to have a constant water depth. To illustrate the 3-D effect of the wedge, Figure 3 and Figure 4 include an equivalent calculation for a flat bottom. The offsets needed to evaluate equation (1) for a flat bottom are derived by inspection of Figure 5.

|  |  |  |
| --- | --- | --- |
|  |  | (4) |

|  |  |  |
| --- | --- | --- |
|  |  | (5) |

where

= range and depth of original source relative to ocean surface;

= range and depth of each receiver relative to ocean surface;

= cross-slope distance of the receiver relative the vertical source/origin plane;

= depth of each source image;

= water depth;

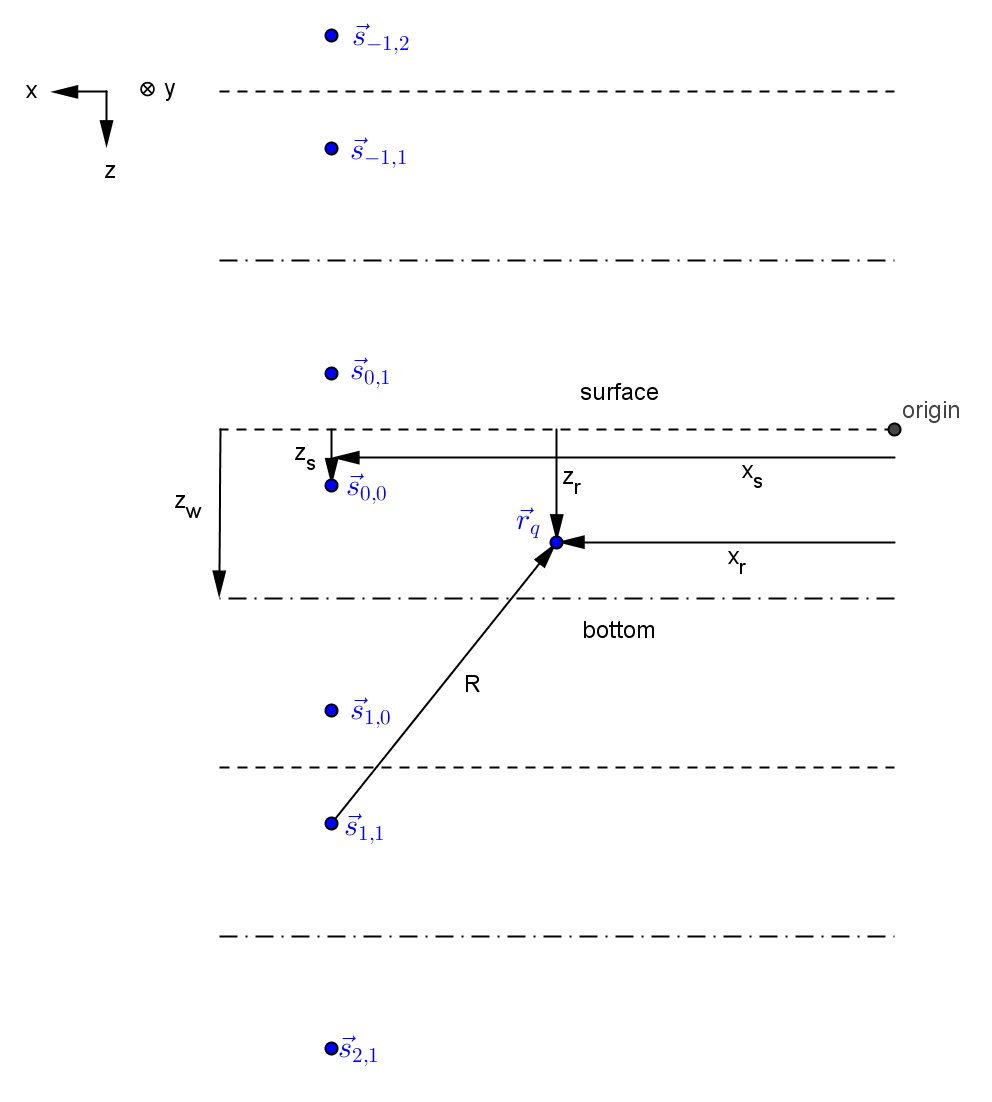


Figure 5 – Method of images for flat bottom

Results for more realistic slope angle are illustrated in Figure 6. The realistic scenario is almost identical to the 25 Hz ASA wedge benchmark [3], but the bottom reflection coefficient has been hard coded to +1. Figure 7 illustrates the simplified ASA wedge scenario at a frequency of 250 Hz, a frequency close to the lower bound of WaveQ3D accuracy.

= 4001.2 meters

= 2.86o

= 1.43o (middle of water column)



Figure 6 – ASA wedge transmission loss as function of cross-slope range at 25 Hz



Figure 7 – ASA wedge transmission loss as function of cross-slope range at 250 Hz

Both of the 20o and 2.86o scenarios have significantly stronger losses as a function of distance across the 3-D wedge than the cylindrical spreading predicted by an equivalent 2-D model. At long ranges, the 3-D wedge turns high angle paths toward deep water so that they fail to contribute to the received level. The 3-D environment exhibits a cut-off range beyond which no significant amount acoustic energy propagates. The cut-off range increases as frequency increases. These effects grow more pronounced as the angle of the wedge increases.

# WaveQ3D comparisons to analytic solutions

This evaluation uses a simplified version of the ASA wedge benchmark.

= 21.0375o

= 520 meters

= 100 meters

= 200 meters

= 1500 m/s

Because the WaveQ3D calculations are performed in geodetic coordinates, the simple wedge used in our analytic solution can only be approximated in WaveQ3D. On a round Earth, an interface with constant slope is a curved surface instead of a plane. To minimize the impact of this curvature, the source and receivers are placed at a depth of 100 meters at the Equator. The water depth at this point is set to 200 meters and the bottom slope is a constant 2.86o, sloping down to the north, at all latitudes and longitudes. This definition orients the wedge illustration in Figure 1 such that the x-direction is north, the y-direction is east, and the z-direction is down. Receivers are placed east of the source, along the y-direction, at varying cross slope ranges.



Figure 8 – Compare coherent WaveQ3D to analytic solutions



Figure 9 - Compare incoherent WaveQ3D to analytic solutions



Figure 11 - Top down view of ray paths for -10o D/E



Figure 12 – 3-D view of individual ray path for -10o D/E and 135o AZ

# Works Cited

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| [4] | L. Felsen, "Numerical solutions of two benchmark problems," *J. Acoust. Soc. Am.,* vol. 81, no. Suppl. 1, pp. S39-S40, 1987. |